Genetic Algorithms and an Exploration of the Genetic Wavelet Algorithm

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Abstract

Genetic Algorithms (GA) have been a branch of Artificial Intelligence since the mid 1970's. Since then, many different kinds of GAs have been invented; however, most of these genetic algorithms are a crude representation of the evolutionary mechanisms from which they model. The Genetic Wavelet Algorithm is an Evolutionary Algorithm developed by Jeffery Freeman of Syncleus, Inc. that attempts to more accurately model the evolution than the traditional GAs. The purpose of this lecture is formally to define the Genetic Wavelet Algorithm and describe how what is required for it to be implemented. In addition, the performance of the Genetic Wavelet Algorithm will be compared to the classic Simple Genetic Algorithm on difficult instances of NP-Hard problems.
1. Evolutionary Algorithms

1.1 Background

Evolutionary Computing is the branch of AI which consists of optimization problem solving techniques that originate from the abstraction of evolutionary principles found in biology. The part of Evolutionary Computing that the Genetic Wavelet Algorithm stems from is Evolution Algorithms; the group of algorithms that are deemed Evolutionary Algorithms (EAs) are biologically inspired and use simulated evolution to drive a search process. EAs use simulated genetics to solve problems.

Artificial intelligence (AI) "was proposed for the first time by John McCarthy in 1956, when organizing a conference at the Dartmouth College on intelligent machines."[6] The first Evolutionary Algorithms were seen soon after the birth of the field of Artificial Intelligence. The first formal research on Evolutionary Algorithms was introduced in Lawrence J. Fogel's Phd dissertation in 1964. A few years later in 1966, Fogel along with Alvin Owens and Michael Walsh
published a book on Evolutionary Programming. The foundation of EAs was expanded in 1973 by Ingro Rechenberg in his paper about evolutionary strategies.[3] In 1975, John Holland published the first literature on what he called Genetic Algorithms. Since then the field of Evolutionary Computation has expanded to a wide variety of applications, all stemming from the biological concept of evolution.

In order to better understand the fundamentals of EAs, one must understand the biological concepts driving them. In 1866, Gregor Mendel was the first to formally describe genes and heredity in what is called transmission genetics. After breeding experiments with the plant Pisum sativum, he found that some observable traits were controlled by what we now call genes, which were inherited independently from other genes. He also discovered that adult organisms carry two copies of each gene, one copy is inherited from each parent. One of these sets of genes is known as a chromatid, and both are referred to as the individual's chromosome. Modern understanding of genetics tell us that an organism's chromosome is stored in its Deoxyribonucleic acid (DNA), and it holds all the genetic information of the organism. DNA is a microscopic polymer made out of repeating pairs of nucleotides - the basic building blocks for DNA. A
gene consists of multiple pairs of nucleotides, and corresponds one or more regions of the genome. An allele is an "alternate form of a gene."

So when Mendel first observed the different colors of pods of Pisum sativum plant, the diversity he was observing was caused by different alleles. A visible characteristic of an organism, or phenotype, could be caused by multiple pairs of alleles. Because organisms have two complete sets of genes, the phenotype expressed by a specific gene results from the allele on each side of the chromosome. When these two allele are the same, the gene is homozygous. When the alleles are different, the gene is heterozygous. Sometimes in heterozygous alleles, one allele is dominant and the other is recessive, causing the phenotype to be dictated by the dominant allele. For other genes, heterozygous alleles causes a phenotype that shares the phenotypic properties of both alleles.[14]

The other largely influential biological idea that EAs abstract is natural selection. The idea of natural selection was introduced by Charles Darwin in 1859: “Owing to this struggle for life, variations, however slight and from whatever cause proceeding, if they be in any degree profitable to the individuals of a species, in their infinitely complex relations to other organic beings and to their physical
conditions of life, will tend to the preservation of such individuals, and will generally be inherited by the offspring. The offspring, also, will thus have a better chance of surviving, for, of the many individuals of any species which are periodically born, but a small number can survive. I have called this principle, by which each slight variation, if useful, is preserved, by the term Natural Selection.”[15] Darwin recognized the profound ability nature has to eliminate inferior species and promote the more fit ones. This property of life has yielded successful results (living species after an estimated billion years of the existence of life), and it is that same property that was abstracted and simulated in EAs.

Darwin's observations can be summarized by three principles: "There is a population of individuals with different properties and abilities. An upper limit for the number of individuals in a population exists. Nature creates new individuals with similar properties to the existing individuals. Promising individuals are selected more often for reproduction by natural selection."[3] There is one thing that Darwin did not quite accentuate here, and that is the role of mutation. Mutation is when the genetic information of the offspring is not identical to either of the originating parents. Mutation occurs for many reasons, and can be both harmful and helpful to the offspring.
depending on where in the DNA the mutation occurs. When a mutation is especially helpful to a species, it becomes a "promising individual," therefore the trait will be propagated to future generations. Essentially, mutation is responsible for new diversities in a population, making it an essential part of EAs.

1.2 Properties of EAs

Evolutionary Algorithms come in many different varieties, but in order to be classified as an EA, an algorithm must have certain basic properties. According to the Handbook of Evolutionary Algorithms, there are three integral properties shared amongst all EAs:

1. Population: "Evolutionary algorithms utilize the collective learning process of a population of individuals."[1] The population is essentially a group of possible solutions generated by the algorithm - all of which are evaluated and then the best are chosen.

2. Reproduction and Mutation: "Descendants of individuals are generated by randomized process intended to model
mutation and recombination."[1] Mutation happens when an individual erroneously self-replicates; this is done purposely and is important for ensuring diversity of individuals amongst a population. Recombination is the reproduction step, two or more individuals are combined in order to distribute their individual information. The recombination step is usually how new individuals are introduced into the population.

3. Evaluation: "By means of evaluating individuals in their environment, a measure of quality or fitness value can be assigned to individuals."[1] This measurement is usually done by a fitness function; from an evolutionary standpoint, this represents the environment. A fitness function is the part of the algorithm that drives the algorithm. Without a form of evaluation, the differences between individuals would be indistinguishable.

In short, an EA is an algorithm that searches through a population of individuals. Individuals are evaluated, mutated and then recombined. The process is simple, but fine details go into any implementation of an EA. EAs are a powerful tool in computer science, but traditionally have their limits. Some examples of EAs include:
Genetic Algorithms, Genetic Programming, and Evolutionary Programming. The Genetic Wavelet Algorithm (GWA) applies as an Evolutionary algorithm, because it also follows these three paradigms.

The process for Evolutionary algorithms is diagrammed below. The population is initialized, mutation occurs (most likely not in the first iteration), each individual in the population's fitness is evaluated, then crossover and repeat the cycle until stopping conditions are met. The cycle of steps is simplistic, however it is not trivial.

![Evolutionary Algorithm Flowchart](image)

Fig. 1.1 – Evolutionary Algorithm Flowchart [4]
1.3 EA Uses and Limitations

There are many questions a developer should ask when implementing an EA; for instance: how should one represent the individuals of the population? What should the population size be? How should the population be updated after selection is applied? How should mutation affect an individual? When should the algorithm stop? The answers to these questions is different for certain type of EAs, and is also dependent on the type of problem being solved.

Before addressing the above questions concerning implementation details of EAs, there is a simpler question that needs to be answered: Why Evolutionary Algorithms? EAs are essentially a form of search. So, what sets it apart from other approaches when considering how to solve a problem? Some advantages include: "simplicity of approach, its robust response to changing circumstances, and its flexibility."[4] Holland argued that the power of EAs (more specifically GAs) comes from their ability to solve problems in an "implicitly parallel fashion."[16] Part of the flexibility of EAs is the wide variety of problems that can be solved. If a problem can be formulated as a functional optimization problem, then it can be solved by EAs. In addition, EAs have the ability to solve problems that have not been
solved. "Fogel (1995) declared artificial intelligence as 'They solve problems, but they do not solve the problem of how to solve problems.' In contrast, evolutionary computation provides a method for solving the problem of how to solve problems."[4]

EAs are not the answer to every problem, because they have some problems of their own. Some optimization problems lead EAs to false solutions, when the algorithm finds a locally optimal solution that meets the stopping criteria. Locally optimal solutions represent roots (local maximum or minimum points in the search space), and to an EA, "one root is as good as another."[5] Highly nonlinear functions are also difficult for EAs to optimize, partly due to a greater occurrence of locally optimal solutions. "Typical approaches to highly nonlinear problems involve either linearizing the problem in a very confined region or restricting the optimization to a small region. In short, we cheat."[5] Another limiting factor of EAs is the representation of an individual's genome. For example, If it is chosen to be a fixed length, then that is a limiting factor for the solution - if the optimum solution does not fall within the representation's range of solutions, then the optimal solution will not be found. Another disadvantage of EAs is setting one up to solve a problem. A great amount of understanding of
a problem is required to know how to represent a solution, tests its fitness and have an effective termination condition.[10]

This paper approaches the topic of Evolutionary Algorithms first by providing an understanding of existing genetic algorithms and then relating the paradigms found in GAs to the formal definition of the Genetic Wavelet Algorithm.
2. Classic Genetic Algorithms

"An algorithm is a series of steps for solving a problem. A genetic algorithm is a problem solving method that uses genetics as its model of problem solving."[4]

2.1 Introduction to the Classic GA

Genetic Algorithms (GAs) are a common example of evolutionary algorithms. There are many different variations of GAs. A classic or simple GA represents the solution with a fixed length string, usually a bit string. GAs use recombination and mutation to generate the next iteration of a population. "The objective function or fitness function f(s) plays the role of the environment; each individual s is evaluated according to its fitness. In this way a new population (iteration t+1) is formed by selection of the better individuals of the former population, as they will form a new solution by means of applying selection procedure and crossover and mutation operators. It should be noted that diversity of individuals is required to find good solutions with GA."[2] The steps of simple GA are outlined below.
Above is a flowchart similar to the example given for EAs. In this flow chart, the generic steps for EAs are expanded and detailed to show the more specific process of a simple GA. The first step is the initialization of the problem's fitness function, which is referred to as cost in the flowchart because it applies to a minimization problem. An initial population is generated (usually randomly). Next, the algorithm simulates generations of individuals by using the genetic operators (crossover and mutation) until the stopping criteria is met.[5] The logical steps for GAs are a little less simplistic than in EAs and are not defined well enough in the diagram above.
Before the algorithm can start, the encoding of the chromosome has to be chosen. The encoding will be a fixed length string, therefore any possible solution to the problem has to be able to be encoded in the chromosome. That means that the scope of the problem (or at least the desired solution space) has to be defined, and the chromosome has to have the capacity to represent it.[4]

Next, the fitness function is defined. The fitness function needs to enforce all the constraints and objectives of the problem. Any constraint or objective "can be handled as weighted components of the fitness function."[4] Therefore, if a particular individual violates a constraint then the fitness value should decrease, and if it satisfies an objective then the fitness value should increase. The amount of increase/decrease is dependent on the weight of the constraint or objective. The weight signifies the importance of each constraint relative to another constraint.

After the parameters (encoding and the fitness function) of the GA are set up, an initial population is created. The population size should be large enough to ensure diversity amongst individuals, and is also dependent on the nature of the problem. An initial population with the greatest diversity amongst individuals is most beneficial for
avoiding locally optimal solutions and convergence. In classic GAs, the initial population is usually created randomly. Creating individuals randomly will provide enough diversity in larger populations, but can potentially initialize very similar individuals, especially in smaller population sizes.[4]

The step of decoding the chromosome breaks down the genome representation into parts that can be used by the fitness function. Usually this involves breaking down the chromosome into alleles, each of which represents a piece of the solution to be tested by the fitness function.[5] It is the sum of these components which dictates an individual's fitness. Each individual is tested, and a group is chosen to breed the next generation.

The selection of which individual mates with another can be decided in many ways. The roulette wheel is a popular method in which individuals are weighted by their fitness and randomly chosen. [4] In this case, ones with higher fitness have a higher probability of getting chosen. Another method is to mate the most genetically dissimilar individuals. This promotes diversity and convergence. This is especially easy when using bit strings as representation.
2.2 Mating and Crossover

Mating is the step of a GA in which the cross over operator is applied to the selected mates to form a new individual. This step is essentially concatenating substrings - one from each mate, the point at which the string is split is called the crossover point. The placement of the crossover point is dependent on how alleles are represented in the chromosome string[4]. Taking parts from each parents string representation and putting them together creates a new individual, but how should these new individuals be put into the population? The new generation could replace the old one completely, or some of the best performing individuals could be kept in the population with the new generation. Poor performing individuals should be removed from the population. If the older generation is entirely replaced by the new one, the algorithm runs the risk of backward progress (the new generation performing worse than the old one). Below is an example of crossover, bit strings a and b combine to create the two bit strings c and d. Notice that this is very unlike the way genetic information is passed to offspring in the biological context.
Every GA has a mutation factor or mutation rate, which is just a number that represents how large of a portion of the genome is mutated for each individual. When mutation occurs, an individual becomes more or less fit. The higher the mutation factor is, the less powerful the crossover step is. However, if the mutation factor is low (e.g. 1 bit in a bit string of length 1,000,000,000) then the population will take more generations to generate diversity that does not already exist in the population.[10] Mutation affects the chromosome randomly; how it is applied is dependent upon how the genome is represented. For instance when using a bit string encoding, bits are flipped, but when using integers or real values mutation takes the form of addition or subtraction.
The steps of evaluation, recombination and mutation are performed until some stopping criteria is met. In genetic algorithms the stopping criteria is convergence, meaning that the population has converged towards a single solution or solution set.[4] Sometimes the problem is such that if an individual reaches a certain fitness, the answer is found. Convergence is measured by the most fit individuals in consecutive generations. If the most fit individual is not improving over many consecutive generations, then the algorithm has come to a point of convergence. Convergence does not mean that the solution has been found, but that a solution has been found. The solution could be found to be locally optimal, in which case another constraint would need to be added to the fitness function.

How does one encode diversity into these algorithms? Not diversity amongst its population's individuals' chromosomes, but the diversity that can be found by comparing a flea to a blue whale. It is not easy, GAs chromosome representation is inherently discrete, because they are fixed length strings. Fixed length means a predefined minimum and maximum, making the scope of the individuals limited - falling short of mimicking biological evolution's ability to adapt new unseen traits in response to an environment.
3. Chromosome Representation for GAs

3.1 Importance of representation

"The coding of the variables in [binary] string structures make the search space discrete for GA search. Therefore, in solving a continuous search space problem, GAs transform the problem into a discrete programming problem. Although the optimal solutions of the original continuous search space problem and the derived discrete search space problem may be marginally different (with large string lengths), the obtained solutions are usually acceptable in most practical search and optimization problems. Moreover since GAs work with a discrete search space they can be conveniently used to solve discrete programming problems, which are usually difficult to solve using traditional methods." GAs are diverse and capable of solving difficult problems that non-evolutionary search algorithms have difficulty solving. Encoding the solution in bit strings has its limits. This limit has been recognized by computer scientists, and other techniques of representation have been developed for removing some of these limitations.
The way a GA represents an individual's chromosome is important for understanding its limitations and application to classes of problems. On some level, any chromosome is represented by a bit string, because that is how the computer understands it; however, what is important is how alleles are represented, because they are the smallest logical element of a chromosome in GAs.

### 3.2 Approaches to chromosome representation

Heinz Muhlenbein says that GAs can be broken down into three general approaches to chromosome representation: phenotypic, genotypic, and statistical.[2] Phenotypical approach is more concerned with the actual observable behavior of an object than its specific genes. An example of a phenotype would be the color of someone's eye - there are many genes that go into affecting this, but it is observed as a single characteristic. Phenotypic approaches correlate the chromosome representation directly with a phenotype, instead of using genotype to phenotype mapping. An example of this could be a bit string, in which each bit corresponds to whether or not the individual possesses a specific trait. One benefit of this is for abstracting more advanced behavior in individuals, however it is further away from the capabilities of genetic representation found in
"If a phenotypic property of an individual, like its hair color or eye size is determined by one or more alleles, then these alleles together are denoted to be a gene. A gene is a region on a chromosome that must be interpreted together and which is responsible for a specific phenotypic property."[3] The genotypic approach is almost directly opposite of the phenotypic approach. In genotypic chromosome representation, the bit string corresponds to an individuals exact genetic makeup. The genotypic approach requires a genotype to phenotype map, which takes a substring of the genome (representing an allele) and maps it to an observable phenotype. This is a representation more accurate to what actually happens in biology; a being's phenotypes are influenced by its genotype and the environment. Many components of a being's genotype may make up one visible phenotype. In biology, these components are not usually contiguous, but are dispersed about the genome; however in GAs, the substring is usually contiguous.

The statistical approach, unlike the others, takes into consideration the whole chromosome of each of the individual's
parents. For each allele in each parents' chromosome representation, the gene that the offspring will have is determined statistically depending on the values of the alleles. This could be implemented as some alleles being dominant or recessive, or the resulting could be a combination of both alleles - it all depends on the problem being implemented.

### 3.3 Types of Representations

Below is a table summarizing common representation types for chromosome encoding in GAs. Gray coding and unary coding are mentioned in reference to binary strings and are explained below the table.
<table>
<thead>
<tr>
<th>Type of Representation</th>
<th>Example</th>
<th>Benefits</th>
<th>Limitations</th>
<th>About Most common form of representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary</td>
<td>{10110}</td>
<td>Direct and natural way of encoding for optimization problems</td>
<td>Accuracy is limited by length. Some implementations require mapping of phenotypic bits to a genotypic trait.</td>
<td>Binary strings can use binary, unary or Gray encoding.</td>
</tr>
<tr>
<td>Integer</td>
<td>{12,59,45}</td>
<td>Phenotype to genotype map not usually needed, due to higher precision.</td>
<td>Same limitations as binary representation. Not often used, because integer problems can be more easily solved by using schema processing with bit strings</td>
<td>Represents each allele using integers, can hold more genetic data than an equal length bit string.</td>
</tr>
<tr>
<td>Real-valued</td>
<td>{12.135, -1.4231, 0.23}</td>
<td>Good for solving permutation and combinatorial problems.</td>
<td>Possible precision loss, more precision means more memory</td>
<td>Usually used in GAs that have a high mutation factor.</td>
</tr>
<tr>
<td>Messy</td>
<td>{101, 32, 12.332}</td>
<td>Does not depend on the length of an allele for position in genome. Inversion operators can be used without changing allele order.</td>
<td>Requires more memory to represent. Rarely actually needed. Special operators are required for representation.</td>
<td>Can use multiple types of representation. The Allele's position in the genome is maintained by an index paired with the value.</td>
</tr>
<tr>
<td>Direct Representations</td>
<td>{32}</td>
<td>The value directly represents the genotype. Smaller length representation.</td>
<td></td>
<td>The phenotype directly represents a genotype in a direct representation.</td>
</tr>
</tbody>
</table>

Fig. 3.1 – Table of chromosome representation.
Sometimes a problem occurs when using binary strings in GA optimization problems: the "Hamming Cliff." The Hamming Cliff problem occurs when two parents have bit strings that are dissimilar, but close in value.[1] Take the bit strings "10000000" and "01111111", their decimal values are 128 and 127 respectively, which is one digit away. However, their bit strings share no common values. A real world example of this would be if there were two nearly identical people standing next to each other that didn't have any shared DNA. It would not be a problem if it weren't for the offspring these strings would produce. For instance, say that the crossover point were to be after the third digit. Parents with those bit strings would produce offspring that were represented by the bit strings "10011111" and "01100000." The decimal value of these strings are 159 and 96 respectively. The parents are well fit individuals – we assume this because they are chosen to be mated; because their values are close, it can also be assumed that they are near a point of convergence. The resulting offspring have decimal values very far from their parents, therefore delaying convergence. One solution to the Hamming Cliff problem is to use gray coding for bit strings. Gray coding ensures that consecutive numbers have at most one different bit. With gray coding, the same parents could be represented by the bit
strings "11000000"(128) and "01000000"(127). With the same
crossover point, they would produce two offspring identical to
themselves.[5]

Another solution to the hamming cliff problem is unary coding. Unary
coding is a simple alternative to standard bit strings; an integer n is
represented by a bit string with n 1's followed by a 0. The length
of a unary coded string is directly related to decimal value of what it
represents. Therefore the number three would be represented by the
bit string "1110". This might not work well with a fixed length string,
because every bit string would have to be as long as the highest value.

A Schema is a way of representing potential individuals that use
bit string chromosome representation. Schemata are strings that
consist of 1, 0 and a wild card character. Take for example the schema
"1**10*1", "*" is the wild card character in this instance. The number
of non wildcard characters in the dictates the order of the schemata,
for the example given the order would be four. A powerful part of of
GAs is the ability to predict the number of copies of a particular
schema in the next generation of a population. After many
generations, well fit schemata that have been prevalent throughout
many generations are deemed as building blocks. After identifying
these building blocks, the schemata can be combined through crossover to get a more fit schemata. Lower order schemata are more likely to have high fitness, so the building blocks start out small, but with the use of recombination they create larger building blocks which speed up convergence.[4]

Variable length representation is not used in classical GAs. Sometimes variable length encoding is useful for certain problem representations. For example, if the problem solution were to be an unweighted graph that is represented by an adjacency matrix. A variable length solution would represent a graph with any number of nodes and their edges. But how would one go about recombining these adjacency lists so that they create valid structures and even if they do create valid structures, are they closer to a convergence point? One way could be to ignore invalid structures and let competition weed them out. This strategy works as long as the fitness function has a relatively few number of constraints to test, but if there are many highly coupled constraints the existence of invalid children just works to slow the algorithm down. Richard Dawkins thought of a different way of representing complex problems in GAs - biomorphs. "Taking its inspiration from nature, this approach focuses on genotypic
representations that represent the plans for building complex phenotypic structures through a process of morphogenesis."[7] In his book - The Blind Watchmaker - Dawkins describes "biomorphs" as a way of representing the behavior of a complex object (instead of the object itself). Genes dictate some sort of behavior, like development, instead of traits like height. Instead, the height of an object would be determined by the gene that controlled development.[17]
4. Other Types of Genetic Algorithms

4.1 Adaptive Genetic Algorithms

Adaptive Genetic Algorithms (AGA) are GAs that adapt their parameters while running. Such parameters may include population size, crossover probability or mutation probability. The algorithm is responsive to how much the population improves, for instance if it is not improving much, then the mutation rate may go up.[4] The flow chart for this would look almost identical to that of the GA, with the addition of one step - applying the heuristic for regulating the GA parameters. This happens after the population is evaluated, at the end of the GA loop.

By evolving the parameters that are kept constant in traditional GAs, AGAs are trying to overcome some of the problems inherent to GAs. The first of which is locally optimal solutions. If the GA is converging upon a locally optimal solution, the growth rate of population fitness starts to approach zero. When this happens, the GA will adapt by increasing the mutation factor. This will increase diversity in the next generation, which will provide the GA with new individuals that are not stuck on local optima.
4.2 Fast Messy Genetic Algorithms

Fast Messy Genetic Algorithms (fmGA) is a type of GA that uses variable length binary strings as encoding. The principal feature of fmGAs is their use of building blocks "to explicitly manipulate building blocks (BBs) of genetic material in order to obtain good solutions and potentially the global optimum."[4] The fmGA has three stages: initialization, a building block filter phase, and the juxtaposition phase. The parameter of the length of the building blocks is given to compensate for the variable length strings. Initialization starts with a population sizing equation that finds a size large enough to overcome the noise caused by the building block filtering phase. Once population size is established, a population is randomly generated and their fitness is evaluated. Members of the population are then used to derive a building block that is the desired length. This is accomplished by a building block filtering schedule, which constitutes the building block filtering phase. In this phase, a certain number of bits are randomly deleted from each member of the population. This removal of bits is alternated with a tournament-style selection so that only the fittest building blocks get selected. The building block filter phase is completed when every member of the population has the same length as specified in the problem parameters. In the next phase, the
juxtaposition phase, the best building blocks are randomly chosen and cross over is applied - the crossover point is chosen based upon a probability distribution. This step creates individuals whose strings may or may not be larger than the specified building block size, these individuals make up the next generation. These phases are repeated until convergence or finishing criteria are met.[4]

4.3 Independent Sampling Genetic Algorithm

Another variation on the classic genetic algorithm is the Independent Sampling Genetic Algorithm (ISGA). The ISGA has two phases: the independent sampling phase and the breeding phase. "In the independent sampling phase, a core scheme, called Building Block Detecting Strategy (BBDS), to extract relevant building block information of a fitness landscape is designed. In this way, an individual is able to sequentially construct more highly fit partial solutions."[4] The breeding phase employs a similar technique to one described previously in this paper. Essentially the individuals choose their mates. This is done by finding a mate with a similar fitness that is genetically dissimilar from itself. This algorithm uses a population and a fitness function, but only for the purpose of extracting building blocks. These building blocks are combined using an effective strategy
for convergence. This algorithm is powerful for problems with "difficult landscapes."[4] This means that problems that have many local optima, will be overcome by the breeding of dissimilar pairs; it promotes diversity.
5. Genetic Programming

5.1 Background

Genetic Programming (GP) is a specific application of GAs. Instead of a bit string being evolved, a computer program is evolved. It is evolved using the operators: mutation, crossover, and "architecture-altering operations patterned after gene duplication and gene deletion in nature."[4] GP was first introduced by John Koza in his book, *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, published in 1992. GP stems from and is similar to GAs, however has some important differences. Instead of evolving bit strings, GP evolves tree structures that represent a natural grammar parsing of the program's source code. In addition, GAs use fixed length Strings, while GP needs to use trees which have variable lengths. Instead of the search space being defined by the fitness function, GP always searches for a solution in program space, or the set of all programs that can be written by a language's formal grammar. There is no formal logic required in the search, and there
does not need to be an explicit knowledge base in order to find a solution. In addition, there is not a risk of converging onto a local maximum. But how exactly does GP work?

5.2 Genetic Programming Process

It starts by initializing a population of tree structures. The tree structures in GP are organized such that all leaves are terminals (values or variables) and internal nodes represent functions. GP is implemented with a maximum tree height to ensure that programs do not become overly complex. There are two different methods for initializing trees, "called full and grow."[18] Grow starts with a function node at its root, and grows it by a single random node - terminal or function - until all the leaves are terminals. Full generates only random functions until the next node to be generated is at the maximum height, then it generates random terminals. This results in a tree that is full size.[18] Combining these two forms of tree creation provides an initial population of many different sizes.

After the population is created, individuals are evaluated and used for the creation of the next generation of programs. One difference between GP and GAs is that GP keeps some inferior programs, due to the possibility that these programs could evolve into
a more optimal solution. Because GP keeps a population, these inferior programs do not interfere with better performing ones. GP performs crossover similarly to the way GAs do, by using a crossover point. Instead of a place in a string, this is usually an edge in the tree. Two parents produce two offspring, each of which contain complementary parts of the parents. The process is diagrammed below, A and B are the parents that crossover to create children C and D.

```
A
  -
   \-\-
     2.2 / \ 7
       X   11  Y

B
  *
   \-\-
     cos / \-
       7   Y

C
  -
   \-\-
     7 / \-
       X   11

D
  *
   \-\-
     cos / \-
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Evaluating the programs created by the process of GP can be difficult. An implementation of GP needs to have a fitness function that is able to distinguish between the hopeful solutions and the ones that need to be weeded out. Programs may be generated that cannot be compiled. "We can think of this as a form of infant mortality, in the sense that programs that are not able to be successfully compiled never get a chance to be executed."[7] Along with programs that are not executable, some programs will have parts of their code that do nothing to promote their fitness. "This phenomenon has been dubbed 'bloat' in the evolutionary computation community and appears to have rather similar features to biological systems."[7] Bloat in individuals has its own purpose - the bloat part could be modified through crossover or mutation, and become something that makes the individual more fit without affecting the parts that made it fit in the first place.

The end conditions are met when the program produces a certain output or when a convergence point has been found. In a properly configured GP set up, convergence means that GP either found the answer to the problem, or that the problem cannot be
solved with that maximum length program. The important part of GP is evolving a set of instructions. Biomorphs had a similar theme in its implementation.
6. Evaluation of Evolutionary Algorithms

When evaluating any type of EA, specifically GAs, it is necessary to judge the problem which the algorithm is attempting to solve. "when using specific representations some problems become easier, whereas other problems become more difficult to solve for[Genetic Evolutionary Algorithms]."[3] Therefore evaluation should start with judging the problem. If the EA is using a different representation than the easiest representation for the problem, then it is a sign to change the chromosome representation. A few widely used methods for measuring problem difficulty in GAs/EAs are correlation analysis, polynomial decomposition, Walsh coefficients and schemata analysis. [3]

Evaluating a solution is extremely important when dealing with GAs, because of their tendency to find local extrema. Any solution returned from a genetic algorithm should be recorded and written back into the system for further generation. It is important to change the parameters of the GA, such as population size and mutation rate, when putting a found solution back into the GA.[4] In addition, GAs use “metaheuristics” which essentially means that a single GA can generate different solutions for separate runs. “This particular
characteristic of metaheuristics supposes an important problem for the researchers when evaluating the results and, therefore, when comparing their algorithms to other existing ones.”[11]

Comparing similar Genetic or Evolutionary algorithms is not an easy task, and therefore often comes down to an empirical analysis of one algorithms performance compared to another one. These performances are tested over multiple well known problems. The comparison consists of the quality of answer found and the computational effort that is used to find the solution.

The most important part of evaluating the GWA is empirically - by testing it against implementations of some of the algorithms defined above. By comparing convergence on a range of difficult problems, it will be clear if the complexity that the GWA introduces to the classic GA creates a stronger Evolutionary Algorithm.
7. The Genetic Wavelet Algorithm

7.1 Introduction to the GWA

This paper has broadly covered the topics of EAs, GAs and GP. The Genetic Wavelet Algorithm (GWA) has common characteristics with these topics and the foundational information already presented is important in order to understand the components of the GWA. The GWA has a population of individuals that are evaluated by a fitness function, and it uses recombination operators to create new generations of the population, therefore it can be classified as an evolutionary algorithm. However, there are more details that go into the GWA than classic GAs. Below is a flow chart with the basic steps of the GWA.
In relation to other EAs & GAs, the GWA is similar. It follows the basic steps paradigm of population, crossover, and mutation. The large difference comes in the way the GWA represents individuals in the population and their chromosomes. The steps of population evaluation (fitness function), convergence check, population size, and the step of mating are all the parts that need to be implemented when solving any problem. Before implementation, one must know the details of the GWA. The steps and specifications are explained below.
7.2 Population Initialization

Individuals in the GWA are represented as organisms. An organism is made up of one or more cells. A cell has a nucleus which contains one or more chromosomes. There can be single-cellular implementations of the GWA or multi-cellular. An example of a multi-cellular implementation would be a neural network.

The implementation of the organism is dependent on the problem and how the programmer wants the genes to be represented in the algorithm. This includes whether it is going to be a single-cellular or a multi-cellular implementation. This organism also is where genes would be mapped to traits.

Regardless of how an individual is implemented, a cell is initiated the same way. A cell starts off with one chromosome, and a mutability factor, a floating point variable between zero and ten. A cell can have more chromosomes if mutation occurs; when it is instantiated, a random number (between zero and ten) is tested against its mutation factor, if the random number is less than the mutation factor, another chromosome is added. This continues until a random number greater than the mutability factor is chosen.
The GWA represents chromosomes as pairs of chromatids. Each chromatid has a set of genes. There is a right chromatid and a left chromatid. They are joined at what is called the centromere position, which dictates where the gene will be split for crossover. Each chromatid is instantiated with its own mutability factor, and then one or more gene is added to the chromatid, depending on the same method above for adding chromosomes to the cell.

The chromosome representation is done as objects in lists, therefore an allele is not dependent on a position in a string, but on an index in a list. At each index of the list is a gene object. There are three types of genes: promoter genes, signal genes, and external signal genes. Each gene has a list containing zero or more receptors. Each gene has an output which is computed by its expression function. This output is a floating point value and is essentially its value, however the output of can also be represented by a set of wavelets, more on this further on. Before additional detail on the types of genes, it is important to discuss the concept of “key.”

The GWA uses keys, which use schemata to encode their values. A key is a variable length string that is evolved during the genetic
wavelet algorithm using x as a wild card character. A key might look like this:

A key is a schema, but it is not used for chromosome representation, instead it is used to model communications between cells in the GWA. Keys are used to represent two important components of the GWA - receptors and signals. The interaction is loosely modeled after hormones binding to cells. There are restrictions for signals binding to receptors. A signal can bind to a receptor if and only if the receptor matches the signal on all non-x points. For example, the signal key "01101x10" can bind to the receptor "110xx10," but "0101xx1" cannot bind to the same receptor.
Keys do not have to be equal in length in order to bind to one another; a smaller length receptor would bind with more signals, because it would bind to more sites on a signal. These signal and receptors are an integral part of the GWA. Keys can mutate along with the system, however they are not changed on crossover. A key is initialized with one or more values depending on the mutation factor of the gene that is initializing it. All of the genes in the GWA use keys in some way.

There are three types of genes; they all have a few things in common. First, they all have one or more receptors. Receptors have a receptor key. A gene starts off with at least one receptor, depending on mutation. Every gene also has an expression function, as
mentioned before. The expression function is calculated using the receptors to map each one to a dimension in a wavelet. More details on how the expression function is calculated can be found in section 7.3. The floating point value output of the expression function is known as the current activity of a gene – it is the value that it outputs.

The first type of gene is the signal gene. Signal genes are responsible for creating a signal key that may or may not bind to other genes' or its own receptors. Signals genes each have a concentration which is calculated in the tick step. A signal gene outputs its signal local to the cell it resides in. A signal binding to a receptor will increase the output of that receptor by its concentration in the affected gene’s expression function.

An external signal gene is similar to a signal gene, but it has a direction that it is facing – it can either be inward facing or outward facing. If it is outward facing, it outputs signals to other cells, and its receptors receive signals from within the cell. If it is inward facing, then it outputs signals to the local cell, and receives signals from other external signal genes in other cells. These genes are only used in multi-cellular implementations.
The final type of gene that is used in the GWA is a promoter gene. Promoter genes exist to promote a gene that is a certain distance away from it on the same chromatid. Promotion of a gene is calculated and affects the current activity of a gene. The current activity is increased by the product of its expression function output and how much it is being promoted. The gene that a promoter gene affects can change due to mutation.

The step of population initialization starts with filling the population with randomly generated individuals. Each one of those individuals is created with one or more cells, each containing one or more chromosomes. Each chromosome is initialized with one or more gene, which could be a signal, external signal, or promoter gene.

### 7.3 Pre-Tick, Tick, Wavelets, and the Expression Function

The pre-tick step is the first step in the simulation phase. This is where each gene calculates its new activity from its expression function. This is stored as its pending activity, because it is not applied as its expression yet. Each signal that is being expressed in the cell is tested to see if it binds to each receptor on the cell. If it does bind to a cell, then its value is changed, if not its value is zero.
This step in the simulation calculates and applies the promotion value. The current activity is set to the sum of the pending activity and the product of the pending activity and its gene’s promotion. Promotion does not affect value of the expression function, just changes its output.

The expression function is the core of the GWA, and the origin of the algorithm’s name. The expression function is a collection of one or more wavelets. A wavelet is a wave that starts out with amplitude of zero which increases and then goes back to zero. A common example of a wavelet is the wave caused by a heartbeat. A wavelet is described by a few different properties: amplitude, center point, phase, form, and distribution. When a gene is initialized, the expression function is created with one initial wavelet that is generated randomly. The number of wavelets in the expression function is dictated by mutation. When a gene mutates, its expression function has a chance to add, remove, or modify wavelets. Below are two examples of what the expression function could look like. The first is a two dimensional one (two receptors) with one wavelet, the second one is what it would look like if another random wavelet were to be added to it.
Fig. 7.2 – Wavelet examples

The expression function is multi-dimensional; each dimension corresponds with a receptor, and the receptors value. A receptors value is zero, unless a signal binds to it. Each wave in the expression
function will have the same dimensional values. So, in the above examples you see each wave has two dimensions – x and y. Therefore if these waves were expression functions, the genes that they were from would only have two receptors on them. Or they could have twenty receptors and only have two of them actually be bound to signals.

The expression function can be used in two ways, through its floating point output, and as a single function that it can represent. Because the expression function is a function, it can be used as part of the solution. For instance, by using a technique called convolution, multiple wavelets can be combined into a single wavelet; using the technique on the two wavelets above result in the wavelet below.

![Wavelet transform example](image)

**Fig. 7.3 – Wavelet transform example**
Using a gene’s expression function as a way to map the organisms’ behavior depending on different input could yield powerful results. Again, the way the genes affect an organism is up to the details of the problem and its implementation. Either way, the expression function’s method of adding multiple multi-dimensional wavelets is an effective way of building a diverse function. The algorithm can also be implemented by using the expression function’s floating point output.

The way the expression function finds the floating point output corresponds with the multiple wavelets. The floating point output comes from the sum of signal concentrations from each individual gene. The output for each wavelet collapses the wavelet; the formula for the collapse is below.

Where \( a \) is the amplitude of the wavelet; \( d \) is the distance from the center (the sum of each dimensional value’s distance from center on that dimension); \( f \) is the form of the wavelet; \( p \) is the phase of the wavelet; \( r \) is the distribution of the wavelet. Let \( c = a \sin(2\pi(d + p/360)) \). The floating point output for the wavelet is \( o \), and \( o = d(c/|c|) (|a(|c/a|^r)|) \).
Whether using the floating point output or the wave function output, the expression function is what is used to calculate the values of each gene depending on the signals that exist in a cell, and the receptors that those signals bind to. This process was modeled off of Gene Regulatory Networks, which is the structure by which organisms propagate their genes.

### 7.4 Evaluating the population and Convergence Check

The fitness function can be implemented like that in any other GA, and it will differ greatly depending on the problem that is trying to be solved. Each gene on the GWA has two potential outputs that could be used in the fitness function as mentioned above. So it needs to be decided whether the implementation is going to be dealing with graphs or values. The evaluation step starts with going through each individual, and evaluating them. Evaluation has to do with how well their genes perform in the fitness function. These details are dependent upon the problem being solved by the algorithm. The convergence check is also dependent on the implementation; traditional checks for convergence were outlined in previous chapters.
7.5 Mate and Mutate

Compilation is the process where two virtual organisms mate to produce a new phenotype belonging to its child. This process is modeled partly after Meiosis in biological organisms. First each parent selects one chromatid from each chromosome at random to pass on to the child. Next each sister chromatid from each parent, pair up at the centromere to reform a chromosome as a composite from each parent. In the end each child will have exactly half of its chromatid from each parent. In addition, each child will have exactly the same number of chromosomes as either parent. Through this process of compilation new children organisms can be formed from mating parents.

Of course all this assumes no mutations occur. Due to mutation it is possible for the chromosome count, chromatid count, and gene count can all differ significantly. Since mutations are usually rare populations should have time to normalize to ensure that such variation will be in the minority.

The mating step is another item that needs to be implemented, but it is not necessarily dependent upon the problem. The crossover
operator is already defined for chromosomes. What needs to be defined is which individuals will mate with other individuals. When this occurs, mutation in the genes is simulated using each chromatid’s mutability factor. The number of chromosomes can also be mutated based upon the mutation factor in each cell of the organism.

Which individuals get chosen for the next generation of the population also needs to be decided by whoever is implementing the algorithm. There are many methods of choosing this that have been described in previous chapters, any of which could possibly be implemented.

7.6 Evaluating the GWA

This algorithm was recently invented; therefore it has not been critically compared against existing genetic algorithms. Therefore its performance compared to other GAs on multiple different types of problems must be empirically examined. The important things to compare are: the number of generations it takes to converge, the number of local optima found, and the evolvability of each algorithm. Evolvability describes the ability of a genetic algorithm to come up with complex novel solutions to a problem; this is best tested in a
problem with no solution, possibly something based on behavior in a stochastic environment.

There are some parts of the GWA that could be evaluated by other metrics. The first of which is its use of schemata – how does the way the GWA use schemata relate to building blocks or the implicit parallelism of schemata that Holland talked about? Also how does it perform when using the expression functions wavelet output to solve problems that have very linear problem spaces? Are there things that could be modified or improved with the current design? All these questions are important for proving the validity, finding the strength, and know the place of the Genetic Wavelet Algorithm.
8. Evaluation of the Genetic Wavelet Algorithm

8.1 Introduction

Both empirical and theoretical evidence is needed in order to accurately measure the GWA’s strength as a genetic algorithm. The empirical test is comparative in nature, testing the GWA against a simple genetic algorithm (sGA) with a fixed-length chromosome representation and floating points as genes. In order for the empirical data to be accurate, the algorithms need to be compared on a set of problems with varying complexity and difficulty to solve.

8.2 Multiple Knapsack Problem

The Multiple Knapsack Problem (MKP) is an optimization problem; given \( n \) knapsacks of a certain capacity and \( k \) items each item having a size and a cost, fill each knapsack to the greatest cost
per size by placing items in them without the sum of the sizes of the
items in a knapsack exceeding the capacity of the knapsack.

The MKP is in the class of problems called NP-hard[19] – which
are the hardest of the problems that take exponential time to find the
optimal solution. This is a good problem to be solved by Genetic
Algorithms, because it can be easily encoded into a chromosome, and
has a simple measure of success – how full the knapsacks are. The
combinatorial nature of this problem makes it difficult to find an
answer using traditional exhaustive search methods, especially as $n$
and $k$ increase.

Genetic algorithms are not guaranteed to find an optimal
solution for every instance of the MKP. The comparison will be
between the solutions produced by the GWA and sGA on the same
instance of the MKP. The most fit individuals will be compared in
intervals of generations. The individuals that the algorithms converge
to and at which generation that individual was initially produced will
be compared. These metrics will show which algorithm gets a more
accurate solution faster and the diversity between generations.

It is important to understand what instances of MKP are
considered harder to solve than others. Testing on random sets of
evenly distributed data does not guarantee a hard problem. The
knapsack problem is considered to be one of the easier of the NP-hard problems to approximate. There is a good measure of how hard an instance of the problem will be. "The problems become harder as the data range is increased." [19] The data range is the distribution of the item values. In problems where items are represented as integers, the valid data range is the size of the largest knapsack. The difficulty of the problem can also be determined by its ratio of number of items and the data range, a value above 1000 is most likely going to be a hard problem. [19] The larger the ratio, the harder the problem.

To compare the two algorithm implementation, the algorithms are tested on many MKP instances with varying degrees of hardness, where the item size to data range ratio is 100, 500, 1000, 2500, and 5000; Comparing the two algorithms on many problem instances at those difficulty levels as well as once with a random distribution, will provide reliable results that can be used to conclude the effectiveness of the GWA on optimization problems compared to the sGA.

8.3 sGA Implementation of Multiple Knapsack Problem

Using the dANN Java AI framework, a simple genetic algorithm using fixed length chromosomes was implemented. The length of the
chromosome is the same as the number of items in the instance of the MKP that it is solving. The chromosomes use floating point values as their genes. One gene represents one item in the list.

The fitness function orders the genes by their floating point values. The function iterates over the ordered list of genes. The size of the item that corresponds with each gene in the list is compared against the remaining space of the current knapsack. If there is not enough room to place the item in the knapsack, the function proceeds to the next knapsack. If there are no more knapsacks, the function ends. The fitness function calculates the weight of the knapsacks by taking the sum of the product each item's size and cost/weight. The cost/weight corresponds to item density, and will always be a double value between 0 and 1 (exclusive). The weight of the knapsacks is then divided by the total space of the knapsack. This is also a double value between 0 and 1 (exclusive), that represents the weight per size of the knapsack. This is a good measurement of fitness, because it represents the value that needs to be optimized in the MKP.
8.4 GWA Implementation of Multiple Knapsack Problem

The current version of the GWA in the dANN library has everything needed for initializing chromosomes and GWA individuals. The population, individuals, mating, and the fitness function were all implemented specifically for the MKP. The GWA, by default, instantiates individuals with one chromosome with one gene on that chromosome. There is a chance that a mutation event will occur and there will be more genes or chromosomes added to that individual. In order to fairly compare the GWA and sGA's performances, the fitness functions should be the same. Because the sGA's fitness function required a fixed length chromosome, and the GWA was implemented to use a fixed length chromosome as well. This is to ensure that there is not an advantage given to either algorithm in their respective fitness functions.

The implementation of the GWA for MKP is constrained. There is only one chromosome is allowed and it is fixed at the number of items in the problem instance. The individuals are single cellular. The goal of this comparison was to test whether or not the underlying mechanisms involved with gene values are superior to the classic sGA. No new signals will be introduced to an individual after it is initialized. The
problem has been constrained as such in order to ensure that the results are such only due to that mechanism.

Mating was implemented as described in the section on mating from the previous chapter. A chromatid was randomly selected from the each of the two parents, combined, crossed over and possibly mutated. After that the child is added to the population and tested. There is also a chance that existing members of the population will mutate on any given generation iteration.

8.5 Results and Comparison

A program was written called Genetic Algorithm Comparator (GAC) to test the two genetic algorithms side by side on the same problem instance. The problem was initiated with a hardness factor described in section 8.2, and a random MKP of that hardness was generated and tested. Both algorithms were run on many sets of problems at different difficulty levels. Each algorithm was initialized with a starting population of 50 individuals, a die off rate of 40%, and a mutability of 20%. After the algorithms solve the same instance of the MKP, their fitness functions are compared and the one with a higher fitness value is considered to be more accurate. The ideal
solution is not calculated, the algorithms’ performances are instead compared to each other. The results are below; each difficulty has two bars, the first one represents the number of problems solved from that particular hardness and each subset represents the number of problems which one of the algorithms has a more accurate solution or if their accuracy is equivalent. The second bar corresponds to the problems which the algorithms were equally accurate, and is split up into sections denoting the number of ties in which the particular algorithm found the solution in less generations. The section labeled TRUETIE refers to finding the solution in the same generation.

Fig 8.1 GWA sGA Comparison Results
The results of comparison are clear, the sGA out performs the GWA on all of the different problem difficulties in terms of getting a more accurate solution. The problem instances in which each algorithm finds the same solution are counted as a tie. The ties are attributed to the algorithm that found the solution in the least amount of generations. The graph below shows the percentage of problems that were more accurately solved by each algorithm.

![Graph representing percentage of problems that each algorithm provided a superior solution for.](image)

The GWA was less accurate than the sGA on 52% of the problems tested. The sGA was less accurate than the GWA on 32% of the problems tested. The two algorithms tied on 16% of the problems.
A tie means the answer’s accuracy is the same, but a tie can be broken by the number of generations that were needed to create that individual. Below is a graph showing the percentages of ties that were broken by the GWA or the sGA. If the two algorithms created the individual on the same generation, then it was considered a true tie. If the ties that were broken were attributed to each algorithm’s accuracy, then the sGA would have been more accurate 54.8% of the time and GWA would have been more accurate 42.72% of the time.

![Graph Showing Percentage of Resolvment of Ties](image.png)

**Fig 8.3 – Graph Showing Percentage of Resolvment of Ties**

The sGA won 18% of the ties encountered in the data sets. The GWA won 67% of its ties, and 16% were true ties. This data suggests that the GWA converges more quickly upon solutions than the sGA. If
the GWA converges faster, then why is it not as accurate? The graph below details the average offset of fitness values between the two algorithms over all the problems.

Fig. 8.4 – Graph showing the average difference between the sGA and GWA's winner fitness when each algorithm has a more accurate solution than the other.
The difference in fittest individual fitness values shows how close a winning individual's fitness was to a losing individual's fitness. For the GWA, the average percent superiority for all levels of hardness is less than 2%. This means when the GWA generates a more accurate solution, it is on average less than 2% higher than the sGA's fitness. On the other hand, when the GWA loses it loses badly. The drastic difference in average fitness begs the question – why is the GWA being outperformed so badly when it loses?

The large gap means that the GWA is getting stuck on local optima. The restricted version of the GWA may be too constrained. The fixed length chromosome had a negative effect on the GWA's ability to diversify. In a non-constrained implementation the chromosomes would be variable length; when a mutation occurred; genes might be mutated, added, or removed from the chromosome. In the fixed length implementation, genes are not added or removed, only possibly mutated. In a population that is converging towards a local optimum, the fitness values of the individuals and their genes will be very similar. Without the ability to introduce new signals or receptors to the gene pool the population has difficulty staying diverse. Below is a chart demonstrating the average generation in which the solution was found for each difficulty set.
Fig 8.5 – Chart demonstrating the average generation in which the final solution was found for the different hardness levels of the MKP.

Despite the fact that the sGA outperformed the GWA on many problem instances, some strengths of the GWA were apparent. The GWA converges more quickly upon solutions compared to the sGA. In the above graph, the average generation to find the solution in for the GWA was approximately eleven. The average generation for the sGA was almost thirty. Even when producing less accurate solutions, the GWA converges upon its answer more quickly. The GWA won 67% of ties, meaning if the sGA and GWA produced the same solution, on average the GWA found the solution more quickly than the sGA. In addition, the small average percentage of superiority of the GWA's
solutions shows that the sGA was very close to GWA's solution. The fact that the GWA was only slightly more accurate also reinforces its strength in converging on a solution. For fitness values that are very close, the solutions that the individuals represent are similar. The GWA was able to converge towards a more specific solution than the sGA, exemplifying its ability to find a more specific answer than the sGA without the sGA being stuck on a local optimum.

8.6 Conclusion

The constrained nature of this implementation of the GWA, specifically the fixed length chromosome, causes the signals in an individual to be less diverse. The lack of diversity causes this implementation of the GWA to be susceptible to converging upon local optima. When not tricked by local optima, the GWA performance proved to be quick and reasonably accurate. The problem constraints existed to test the random valued genes found in the sGA against the weak links between genes caused by signal and receptor interaction found in the GWA. The mechanisms dictating gene values in the GWA are a valid and powerful technology unique to the field of Genetic Algorithms. The code for this implementation is freely available at [http://github.com/koryk/GAC/](http://github.com/koryk/GAC/).
Works Cited


